

## Literature Review of $q$ -Function

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### ABSTRACT

This paper is a literature review of  $q$ -function or  $q$  method. It covers history, background and future applications of  $q$  function. It also contains year wise description of work performed by mathematicians on  $q$  function.

**Keywords:**  $q$  function,  $q$  method, basic hypergeometric function,  $q$  analogue etc.

### I. INTRODUCTION

C. F. Gauss has initiated the theory of hypergeometric series in 1812 and it has been area of research for last two centuries. Numerical Methods are oldest tools of solving mathematical and engineering problems. Numerical computations play an imperative role in solving real time and real life problems of engineering, mathematics and physics. It is an approach for solving complex mathematical problems using arithmetic operations. This approach involves formulation of mathematical models of physical situations that can be solved with mathematical operations. Applications of computer oriented numerical methods have become an integral part of the life of all the modern scientists. The advent of powerful ultra smart computers has tremendously increased the power, speed and flexibility of revised methods of numerical computing.

Increased power of computer hardware has tremendously affected the approach of numerical computing in several ways. Many numerical methods are based on the idea of an iterative process which involves generation of a sequence of approximations with the belief that the process will converge to required solution. Approximations and errors are integral part of human life and they are omnipresent and unavoidable. Data conversion and round off errors cannot be avoided but human errors can be eliminated. Getting perfect solution is main motto of numerical methods but perfection is what we strive for, it is rarely achieved in practice due to wide variety of factors. Reducing error and minimizing number of iterations in numerical method problems is primary concern.

$q$  Method is an extension of classical method by addition of an extra parameter. When  $q$  tends to unity either from right or left,  $q$  function tends to classical function or ordinary function. The theory of hypergeometric function in one or more variables constitutes a natural alliance of much of material discussed by mathematicians from past

three centuries till today. Results on  $q$ -differentiations,  $q$ -integrations,  $q$ -transformations and identities,  $q$ -analogue of certain classical functions and their applications are available in the literature of  $q$  function.  $q$ -analogue finds use in a number of areas, including the multi-fractal measures, study of fractals, expressions for the entropy of chaotic dynamical systems, different integral transforms, differential equations and most effective in boundary value problems and difference equations. It is not like these topics have only been considered by obscure mathematicians of history and it has been a topic of interest to many of the greats of all time, such as Euler, Jackson, Gauss, Jacobi and Ramanujan.

The naturalist, the astronomers, the biologists and the economists all made use of numerical methods. Scope of  $q$  function in numerical analysis pioneered its use in making predictions about eclipses has equally utilized them in its generalization. For an illustration in calculus of variation, weather forecasting, barometrical pressure, force of wind and in genetic coding  $q$  method can be used. Due to diverse application in number of areas of statistics, mathematical physics, chemistry and other fields, great deals of attention have been given to the classical hyper geometric function. Many special functions are also in the form of hypergeometric function.

There is always the possibility of an unlimited number of  $q$  analogues of the most of the special function. Most of the initial examples of  $q$  hypergeometric functions were collected by E. Heine. In last few years Srinivasa Ramanujan was working on applicability of  $q$  function in various problems, which is still a mystery. The topic of  $q$ -special functions is ubiquitous in mathematical Physics and Statistical Mathematics; in particular, they play a basic role in statistical mechanics. Certain  $q$ -series identities have been helpful in proving many combinatorial identities. It is known that Lie algebras play a unique role in the characterization of special functions, and similarly,

quantum groups play the analogous role for  $q$ -special functions. Their significance and wide varieties of applications should not be understated. A number of theorems exist whereby special cases of generalized hypergeometric series can be summed up in closed form. Use of  $q$  function opened the path for the applications to many other fields like crystallography, cosmology, solid state, heat, quantum mechanics and numerical mathematics.

Some of the basic analogues of various summation theorems are as follows.

1.  $q$  Analogue of well poised summation theorem.
2.  $q$  Analogue of Dougall's theorem.
3.  $q$  Analogue of Saalschutz's theorem.
4.  $q$  Analogue of Dixon theorem.
5.  $q$  Analogue of Gauss's second theorem.

In addition to Jackson, various authors for e.g. Rogers, Bailey, Andrews and Watson derived identities of  $q$  hypergeometric series. This list must include the great Indian mathematician S. Ramanujan who gave a number of formulae. Slater undertook systematic investigations of  $q$  hypergeometric identities. One of the mathematicians in the field of  $q$  hypergeometric function who made a significant contribution in the literature of  $q$  method since 1976 is G. E. Andrews. He worked both the analytical point of view, an exemplified by the selection of Slater's output.

## II. HISTORY AND LITERATURE SURVEY

C.F. Gauss started the theory of  $q$  hypergeometric series and worked on it for more than five decades. E. Heine extended this theory and worked on it for more than three decades. Later on F.H. Jackson [Jackson (1904)], [Exton (1983)] in the beginning of twentieth century started working on  $q$  function and proposed  $q$ -differentiation and  $q$ -integration and worked on transformation of  $q$ -series and generalized function of Legendre and Bessel. G.E. Andrews [Andrews et al. (1985)] contributed a lot on  $q$  theory and worked on  $q$ -mock theta function, problems and prospects on basic hypergeometric series,  $q$ -analogue of Kummer's Theorem and on Lost Notebook of Ramanujan. G.E. Andrew with R. Askey [Andrews et al. (1985)] worked on  $q$  extension of Beta Function. J. Dougall [Dougall (1907)] worked on Vondermonde's Theorem. H. Exton [Exton (2003)] worked a lot on basic hypergeometric function and its applications. M. Rahman with Nassarallah worked [Rahman et al. (1985)], [Rahman et al. (1986)] on  $q$ -Appells Function,  $q$ -Wilson polynomial,  $q$ -Projection

Formulas. He also worked on reproducing Kampé and bilinear sums for  $q$ -Racatanad and  $q$ -Wilson polynomial. I. Gessel with D. Stanton [Gessel et al. (1986)] worked on family of  $q$ -Lagrange inversion formulas. D. Stanton [Ismail (2003)] worked on partition of  $q$  series. Theories in the nineteenth century included those of Ernst Kummer [Jackson (1904)], [Exton (1983)] and the fundamental characterization by Riemann of the  $F$ -function by means of the differential equation. Riemann showed that the second-order differential equation for  $F$ , examined in the complex plane, could be characterized (on the Riemann sphere) by its three regular singularities, that effectively the entire algorithmic side of the theory was a outcome of basic facts and the use of Möbius transformations as a symmetry group. L. Carlitz worked on  $q$  inverse relations. R. Y. Denis [Denis (1987)] worked on certain expansion of basic hypergeometric function and  $q$ -fractional derivative and also he published paper on continued fraction.

Subsequently the hypergeometric series were generalised to numerous variables, for example by Paul Emile Appell, but an analogous general theory took long to emerge. Many identities were found, some were quite amazing. Another generalization, the elliptic hypergeometric series are those series where the ratio of terms is an elliptic-function (doubly periodic meromorphic function) of  $n$ .

During the twentieth century this was a prolific area of combinatorial mathematics, with many connections to other fields. There are a number of new definitions of hypergeometric series, by Aomoto, Israel Gelfand and others; and applications for example to the combinatorics of arranging a number of hyper planes in complex  $N$ -space. N.J. Fine [Fine et al. (1988)] also worked on applications of basic hypergeometric function. B.Gorden [Gorden et al. (2000)] worked on mock theta function. I. Gessel [Gessel et al.(1986)] worked on  $q$ -Lagrange inversion formulas. M.D. Herschhorn [Herschhorn (1974)] worked on partition theorem of Rogers-Ramanujan type.

$q$  Series can be developed on Riemannian symmetric spaces and semi-simple Lie groups. Their significance and role can be understood through a special case: the hypergeometric series  ${}_2F_1$  is directly related to the Legendre Polynomial and when used in the form of spherical harmonics, it expresses, in a certain sense, the symmetry properties of the two-sphere or equivalently the rotations given by the Lie group  $SO(3)$  Concrete representations are analogous to the Clebsch-Gordan function.

A number of hyper-geometric function [Jackson (1904)], [Exton (1983)] identities came into light in the nineteenth and twentieth century,

one classical list of such identities is Bailey's list. It is at present understood that there is plethora of such identities, and several algorithms are now known to generate and prove these identities. In a certain sense, the situation can be likened to using a computer to do addition and multiplication; the actual value of the resulting number is in a sense less significant than the various patterns that come out, and so it is with hypergeometric identities as well. M.E.H. Ismail's contribution [Ismail et al. (1977)], [Ismail et al. (1986)] for  $q$  theory is quite remarkable. He worked on  $q$ -Hermite polynomials, biorthogonal rational functions,  $q$ -beta integrals, Contiguous relations, basic hypergeometric functions, orthogonal polynomials and Quadratic birth and death processes and associated continuous dual Hahn polynomials.

Among Indian researchers R. P. Agrawal [Agrawal (1967)], [Agrawal (1976)], [Agrawal (1981)] gave a lot to  $q$  function. He worked on fractional  $q$  derivative,  $q$ -integral, mock theta function, combinatorial analysis, extension of Meijer's G Function, Pade approximants, continued fractions and generalized basic hypergeometric function with unconnected bases. W.A. Al-Salam and A. Verma [Al-Salam et al. (1972)] worked on quadratic transformations of basic series. N. A. Bhagirathi [Bhagirathi (1988)] worked on generalized  $q$  hypergeometric function and continued fractions. V. K. Jain and M. Verma [Jain et al. (1980)] worked on transformations of non terminating basic hypergeometric series, their contour integrals and applications to Rogers Ramanujan's identities.

S.N. Singh [Singh (1987)] worked on transformation of abnormal basic hypergeometric functions, partial theorems, continued fraction and certain summation formulae. K.N. Srivastava and B.R. Bhonsle worked on orthogonal polynomials. H.M. Srivastava with Karlsson [Srivastava et al. (1985)] worked on multiple Gaussians Hypergeometric series, polynomial expansion for functions of several variables. S. Ramanujan in his last days worked on basic hypergeometric series. G.E. Andrews published an article "*The Lost Note Book of Ramanujan*".

H.S. Shukla [Shukla (1993)] worked on certain transformation in the field of basic hypergeometric function. A. Verma and V.K. Jain worked on summation formulas of  $q$ -hypergeometric series, summation formulae for non-terminating basic hypergeometric series in,  $q$  analogue of a transformation of Whipple and transformations between basic hypergeometric

series on different bases and identities of Rogers-Ramanujan Type. B.D. Sears [Sears (1951)] worked on transformation theory of basic hypergeometric function. P. Rastogi [Rastogi (1984)] worked on identities of Rogers Ramanujan type. A. Verma and M. Upadhyay [Verma et al. (1968)] worked on transformations of product of basic bilateral series and its transformations.

In the field of combinatorics [Jackson (1904)], [Extón (1983)] and special functions, a  $q$ -analogue is a simplification involving a new parameter  $q$  that returns the novel theorem, identity or expression in the limit as  $q \rightarrow 1$  (this limit is often formal, as  $q$  is often discrete-valued). Mathematicians are engrossed in  $q$ -analogues that occur naturally, rather than in randomly contriving  $q$ -analogues of predictable results. The primary  $q$ -analogue studied in detail is the basic hypergeometric series, which was introduced in the nineteenth century. M.A. Pathan [Pathan et al. (1979)] worked on bilateral generating functions for extended Jacobi polynomials. R.P. Singhal [Singhal et al. (1972)] worked on transformation formulae for modified Kampe de Ferieet function. M.V. Subbarao [Subbarao (1985)] worked on some Rogers-Ramanujan type partition theorem. C. Adiga and P.S. Guruprasa [Adiga et al. (2008)] worked on three variable reciprocity theorems.

$q$ -analogues find applications [Jackson (1904)], [Extón (1983)] in a number of areas, including the study of fractals and multi-fractal measures, and expressions for the entropy of chaotic dynamical systems. The liaison to fractals and dynamical systems results from the fact that many fractal patterns have the symmetries of Fuchsian groups in general (e.g. Indra's pearls and the Apollonian gasket) and the modular group in particular. The connection passes through hyperbolic geometry and ergodic theory, where the elliptic integrals and modular forms play a prominent role; the  $q$ -series themselves are closely related to elliptic integrals.  $q$ -analogues also came into sight in the study of quantum groups and in  $q$ -deformed super algebras. The connection here is alike, in that much of string theory is set in the language of Riemann surfaces, ensuing in connections to elliptic curves, which in turn relate to  $q$ -series.

$q$  method has a very broad spectrum. It is used in fields like solid state theory, mechanical engineering, operational calculus, quantum theory, cosmology, Lie theory, linear algebra, high energy particles physics, Fourier Analysis, elliptic functions etc.

### III. YEAR WISE DESCRIPTION

Year wise description of work done by various researchers is listed in the table given below.

AUTHOR	TITLE	NAME OF JOURNAL
E. Heine (1847)	Untersuchungen ober die Reiche	J. Reine Angew J. Math.
C.F. Gauss (1866)	<i>Hundest Theoreme uiber die neuen Transscendenten</i>	Werke, Vol. 3, Gottingen
L.J. Rogers (1893)	<i>On the expansion of some infinite products</i>	Proceedings of the London Mathematical Society
L.J. Rogers (1894)	<i>Second memoir on the expansion of certain infinite products.</i>	Proceedings of the London Mathematical Society
F.H. Jackson (1904)	<i>On generalized functions of Legendre and Bessel</i>	Trans. Roy. Soc. Edinburgh Math
F.H. Jackson (1904)	<i>A generalization of the Function (n) and <math>x^n</math></i>	Proc: Roy. Soc. London
F.H. Jackson (1910)	<i>Transformations of q-series</i>	Trans. Roy. Soc. Edinburgh Math
F.H. Jackson (1910)	<i>On q-definite integrals</i>	Quart. J. Pure. and Appl. Math.
F.H. Jackson (1921)	<i>Summation of q-hypergeometric series</i>	Messenger of Math
G.N. Watson (1929)	A new proof of Rogers-Ramanujan identities	J. London Math. Soc.
W.N. Bailey (1935)	<i>Generalized Hypergeometric Series. Cambridge Tracts in Mathematics and Mathematical Physics</i>	Cambridge University Press
W.N. Bailey (1938)	<i>The generating function of Jacobi polynomials.</i>	Journal of the London Mathematical Society
W. Hahn (1949)	<i>Uber Orthogonal polynome, die q-Differenzgleichungen genügen.</i>	Mathematische Nachrichten
L.J. Slater (1951)	Further identities of the Roger-Ramanujan type	Proce. London Math. Soc
L. Carlitz (1957)	<i>A note on the Bessel polynomials.</i>	Duke Mathematical Journal
L. Carlitz (1957)	<i>Some arithmetic properties of the Legendre polynomials.</i>	Proceedings of the Cambridge Philosophical Society
E.D. Rainville (1960)	<i>Special functions.</i>	The Macmillan Company, New York
L. Carlitz (1960)	<i>A note on the Laguerre polynomials.</i>	The Michigan Mathematical Journal
L. Carlitz (1961)	<i>On the product of two Laguerre polynomials</i>	Journal of the London Mathematical Society
L. Carlitz (1961)	<i>Some generating functions of Weisner.</i>	Duke Mathematical Journal
L. Carlitz (1962)	<i>A characterization of the Laguerre polynomials.</i>	Monatshefte fur Mathematik
W.A. Al-Salam (1964)	<i>Operational representations for the Laguerre and other polynomials</i>	Duke Mathematical Journal
W.A. Al-Salam et al. (1964)	<i>Some orthogonal q-polynomials</i>	Mathematische Nachrichten
M.E.H. Ismail (1966)	<i>Relativistic orthogonal polynomials are Jacobi polynomials</i>	Journal of Physics A. Mathematical and General
L.J. Slater (1966)	<i>Generalized Hypergeometric Functions.</i>	Cambridge University Press
L.J. Slater (1966)	Generalized hypergeometric functions	Cambridge University Press, London
R. Askey (1967)	<i>Product of ultraspherical polynomials</i>	The American Mathematical Monthly
L. Carlitz (1967)	<i>The generating function for the Jacobi polynomial.</i>	Rendiconti del Seminario Matematico della Università di Padova
Agarwal et al.	<i>Generalized basic hypergeometric with</i>	Proc. Cambridge, Phil. Soc

(1967)	<i>unconnected bases</i>	
Agarwal et al.(1967)	<i>Generalized basic hypergeometric functions with unconnected bases</i>	Quart. J. Math. (Oxford)
R. Askey (1968)	<i>Dual equations and classical orthogonal polynomials</i>	Journal of Mathematical Analysis and Applications
T.S. Chihara (1968)	<i>On indeterminate Hamburger moment problems.</i>	Pacific Journal of Mathematics
T.S. Chihara (1968)	<i>Orthogonal polynomials with Brenke type generating functions</i>	Duke Mathematical Journal
L. Carlitz (1968)	<i>Some generating functions for Laguerre polynomials.</i>	Duke Mathematical Journal
Verma et al. (1968)	<i>Certain transformations of product of basic bilateral hypergeometric series</i>	India J. Math.
Askey et al.(1969)	<i>Integral representations for Jacobi polynomials and some applications</i>	Journal of Mathematical Analysis and Applications
Askey et al. (1969)	<i>A convolution structure for Jacobi series</i>	American Journal of Mathematics
H.M.Srivastava (1969)	<i>A note on certain dual series equations involving Laguerre polynomials.</i>	Pacific Journal of Mathematics
H.M.Srivastava (1969)	<i>Generating functions for Jacobi and Laguerre polynomials</i>	Proceedings of the American Mathematical Society
G. Gasper (1969)	<i>Nonnegative sums of cosine, ultra spherical and Jacobi polynomials.</i>	Journal of Mathematical Analysis and Applications
R.P. Agarwal (1969)	<i>Certain fractional <math>q</math>-integrals and <math>q</math>-derivatives</i>	proc. Camb. Phil. Soc.
R. Askey (1970)	<i>An inequality for the classical polynomials</i>	Indagationes Mathematicae
H.M.Srivastava (1970)	<i>Dual series relations involving generalized Laguerre polynomials.</i>	Journal of Mathematical Analysis and Applications
G. Gasper (1970)	<i>Linearization of the product of Jacobi polynomials I.</i>	Canadian Journal of Mathematics
G. Gasper (1970)	<i>Linearization of the product of Jacobi polynomials II.</i>	Canadian Journal of Mathematics
Askey et al.(1971)	<i>Jacobi polynomial expansions of Jacobi polynomials with non negative coefficients</i>	Proceedings of the Cambridge Philosophical Society
Askey et al.(1971)	<i>Linearization of the product of Jacobi polynomials</i>	Canadian Journal of Mathematics
G. Gasper (1971)	<i>On the extension of Turan's inequality to Jacobi polynomials.</i>	Duke Mathematical Journal
G. Gasper (1971)	<i>Positivity and the convolution structure for Jacobi series.</i>	Annals of Mathematics
R. Askey (1972)	<i>Positive Jacobi polynomial sums</i>	The Tohoku Mathematical Journal
Al-Salam et al.(1972)	<i>Another characterization of the classical orthogonal polynomials</i>	SIAM Journal of Mathematical Analysis
G. Gasper (1972)	<i>An inequality of Turan type for Jacobi polynomials.</i>	Proceedings of the American Mathematical Society
G. Gasper (1972)	<i>Banach algebras for Jacobi series and positivity of a kernel.</i>	Annals of Mathematics
R. Askey (1973)	<i>Summability of Jacobi series</i>	Transactions of the American Mathematical Society
Srivastava et al. (1973)	<i>New generating functions for Jacobi and related polynomials</i>	Journal of Mathematical Analysis and Applications

G. Gasper (1973)	<i>Non negativity of a discrete Poisson kernel for the Hahn polynomials.</i>	Journal of Mathematical Analysis and Applications
G. Gasper (1973)	<i>On two conjectures of Askey concerning normalized Hankel determinants for the classical polynomials.</i>	SIAM Journal on Mathematical Analysis
R. Askey (1974)	<i>Jacobi polynomials I. New proofs of Koornwinder's Laplace type integral representation and Bateman's bilinear sum</i>	SIAM Journal on Mathematical Analysis
M.E.H.Ismail (1974)	<i>On obtaining generating functions of Boas and Buck type for orthogonal polynomials</i>	SIAM Journal on Mathematical Analysis
G. Gasper (1974)	<i>Projection formulas for orthogonal polynomials of a discrete variable.</i>	Journal of Mathematical Analysis and Applications
M. Rahman (1976)	<i>Construction of a family of positive kernels from Jacobi polynomials</i>	SIAM Journal on Mathematical Analysis
M. Rahman (1976)	<i>A five-parameter family of positive kernels from Jacobi polynomials</i>	SIAM Journal on Mathematical Analysis
M. Rahman (1976)	<i>Some positive kernels and bilinear sums for Hahn polynomials</i>	SIAM Journal on Mathematical Analysis
Askey et al.(1976)	<i>Positive Jacobi polynomial sums</i>	American Journal of Mathematics
Askey et al. (1976)	<i>Permutation problems and special functions</i>	Canadian Journal of Mathematics
Al-Salam et al. (1976)	<i>Convolutions of orthonormal polynomials</i>	SIAM Journal of Mathematical Analysis
Al-Salam et al.(1976)	<i>Polynomials orthogonal with respect to discrete convolution</i>	Journal of Mathematical Analysis and Applications
R.P.Agarwal (1976)	<i>Fractional q-derivative and q-integrals and certain hypergeometric transformation</i>	Ganita
M. Rahman (1977)	<i>On a generalization of the Poisson kernel for Jacobi polynomials</i>	SIAM Journal on Mathematical Analysis
Askey et al.(1977)	<i>Convolution structures for Laguerre polynomials</i>	Journal d'Analyse Mathématique
Al-Salam et al. (1977)	<i>Reproducing kernels for q-Jacobi polynomials</i>	Proceedings of the American Mathematical Society
M.E.H.Ismail (1977)	<i>Connection relations and bilinear formulas for the classical orthogonal polynomials</i>	Journal of Mathematical Analysis and Applications
G. Gasper (1977)	<i>Positive sums of the classical orthogonal polynomials.</i>	SIAM Journal on Mathematical Analysis
H. Exton (1977)	<i>Basic Laguerre polynomials.</i>	Pure and Applied Matematika Sciences
M. Rahman (1978)	<i>A generalization of Gasper's kernel for Hahn polynomials: application to Pollaczek polynomials</i>	Canadian Journal of Mathematics
M. Rahman (1978)	<i>A positive kernel for Hahn-Eberlein polynomials</i>	SIAM Journal on Mathematical Analysis
R. Askey (1978)	<i>Jacobi's generating function for Jacobi polynomials</i>	Proceedings of the American Mathematical Society
Askey et al. (1978)	<i>Weighted permutation problems and Laguerre polynomials</i>	Journal of Combinatorial Theory
C.F. Dunkl (1978)	<i>An addition theorem for some q-Hahn polynomials.</i>	Monatshefte fur Mathematik

T.S. Chihara (1978)	<i>An introduction to orthogonal polynomials.</i>	Gordon and Breach, New York
M. Rahman (1979)	<i>An elementary proof of Dunkl's addition theorem for Krawtchouk polynomials</i>	SIAM Journal on Mathematical Analysis
Askey et al.(1979)	<i>A set of orthogonal polynomials that generalize the Racah coefficients or <math>6 - j</math> symbols</i>	SIAM Journal on Mathematical Analysis
T.S. Chihara (1979)	<i>On generalized Stieltjes-Wigert and related orthogonal polynomials.</i>	Journal of Computational and Applied Mathematics
M. Rahman (1980)	<i>A product formula and a non-negative Poisson kernel for Racah-Wilson polynomials</i>	Canadian Journal of Mathematics
D. Stanton (1980)	<i>A short proof of a generating function for Jacobi polynomials.</i>	Proceedings of the American Mathematical Society
D. Stanton (1980)	<i>Product formulas for <math>q</math>-Hahn polynomials.</i>	SIAM Journal on Mathematical Analysis
D. Stanton (1980)	<i>Some <math>q</math>-Krawtchouk polynomials on Chevalley groups.</i>	American Journal of Mathematics
J.A. Wilson (1980)	<i>Some hypergeometric orthogonal polynomials.</i>	SIAM Journal on Mathematical Analysis
Stanton et al. (1980)	<i>A short proof of a generating function for Jacobi polynomials.</i>	Proceedings of the American Mathematical Society
Stanton et al.(1980)	<i>Product formulas for <math>q</math>-Hahn polynomials.</i>	SIAM Journal on Mathematical Analysis
Stanton et al. (1980)	<i>Some <math>q</math>-Krawtchouk polynomials on Chevalley groups.</i>	American Journal of Mathematics
Verma et al. (1980)	Some summation formulae of Basic hypergeometric series	Indian J. of Pure and applied Math.
Verma et al.(1980)	Transformations between basic hypergeometric series on different bases and Identities of Rogers-Ramanujan Type	J. of Mathematical Analysis and applications
M. Rahman (1981)	<i>A non-negative representation of the linearization coefficients of the product of Jacobi polynomials</i>	Canadian Journal of Mathematics
M. Rahman (1981)	<i>A stochastic matrix and bilinear sums for Racah-Wilson polynomials</i>	SIAM Journal on Mathematical Analysis
M. Rahman (1981)	<i>Families of biorthogonal rational functions in a discrete variable</i>	SIAM Journal on Mathematical Analysis
M. Rahman (1981)	<i>The linearization of the product of continuous <math>q</math>-Jacobi polynomials</i>	Canadian Journal of Mathematics
R.P. Agarwal (1981)	<i>A Family of basic hypergeometric and Combinatorial identities and certain summation formulae</i>	Indian J. pure. Appl. Math
Verma et al. (1981)	Transformations of product of basic bilateral series	Proc. Nat Inst. Sc.
M. Rahman (1982)	<i>Reproducing kernels and bilinear sums for <math>q</math>-Racah and <math>q</math>-Wilson polynomials</i>	Transactions of the American Mathematical Society
Askey et al. (1982)	<i>The Rogers <math>q</math>-ultraspherical polynomials</i>	In: Approximation Theory III, Academic Press, New York
Askey et al.(1982)	<i>A set of hypergeometric orthogonal polynomials</i>	SIAM Journal on Mathematical Analysis
Al-Salam et al.(1982)	<i>On an orthogonal polynomial set</i>	Indagationes Mathematicae
Al-Salam et al.(1982)	<i>Some remarks on <math>q</math>-beta integral</i>	Proceedings of the American Mathematical Society

Verma et al.(1982)	Certain transformations of basic hypergeometric series and their application	Pacific J. Math.
K.K.Baweja (1982)	Transformation of Kampe de Fariet function	GANITA
Al-Salam et al. (1983)	<i>Orthogonal polynomials associated with the Rogers Ramanujan continued fraction</i>	Pacific Journal of Mathematics
Gasper et al. (1983)	<i>Nonnegative kernels in product formulas for <math>q</math>-Racah polynomials</i>	I. Journal of Mathematical Analysis and Applications
Gasper et al. (1983)	<i>Positivity of the Poisson kernel for the continuous <math>q</math> ultraspherical polynomials</i>	SIAM Journal on Mathematical Analysis
Verma et al.(1983)	$q$ -Analogue of a transformation of Whipple	Rocky M.J. of Math.
Verma et al. (1983)	Certain summation formulae for $q$ -series	J. Indian Mathematical Society
Askey et al. (1983)	<i>Recurrence relations, continued fractions and orthogonal polynomials</i>	Memoirs of the American Mathematical Society
Al-Salam et al. (1983)	<i>Sieved ultra spherical polynomials</i>	Transactions of the American Mathematical Society
Al-Salam et al.(1983)	<i>A characterization of the continuous <math>q</math>-ultraspherical polynomials</i>	Canadian Mathematical Bulletin
Gasper et al. (1984)	<i>Product formulas of Watson, Bailey and Bateman types and positivity of the Poisson kernel for <math>q</math>-Racah polynomials.</i>	SIAM Journal on Mathematical Analysis
Rahman et al.(1985)	<i>An infinite series with products of Jacobi polynomials and Jacobi functions of the second kind</i>	SIAM Journal on Mathematical Analysis
R. Askey (1985)	<i>Continuous Hahn polynomials</i>	Journal of Physics A. Mathematical and General
Askey et al. (1985)	<i>Some basic hypergeometric orthogonal polynomials that generalize Jacobi polynomials</i>	Memoirs of the American Mathematical Society
Andrews et al. (1985)	<i>Classical orthogonal polynomials. In: Polynomes Orthogonaux et Applications</i>	Lecture Notes in Mathematics
M.E.H. Ismail (1985)	<i>A queueing model and a set of orthogonal polynomials</i>	Journal of Mathematical Analysis and Applications
M.E.H. Ismail (1985)	<i>On sieved orthogonal polynomials, I: Symmetric Pollaczek analogues.</i>	SIAM Journal on Mathematical Analysis
G. Gasper (1985)	<i>Rogers' linearization formula for the continuous <math>q</math>-ultra spherical polynomials and quadratic transformation formulas</i>	SIAM Journal on Mathematical Analysis
Verma et al.(1980)	Some summation formulae for non terminating basic hypergeometric series	Sian. J. Math. Anal
M.V.Subbarao (1985)	Some Rogers-Ramanujan type partition theorems	Pacific J. Math.
M. Rahman (1986)	<i>A product formula for the continuous <math>q</math>-Jacobi polynomials</i>	Journal of Mathematical Analysis and Applications
M. Rahman (1986)	<i><math>q</math>-Wilson functions of the second kind</i>	SIAM Journal on Mathematical Analysis
Rahman et al.(1986)	<i>A <math>q</math>-integral representation of Rogers' <math>q</math>-ultraspherical polynomials and some applications</i>	Constructive Approximation
Rahman et al.(1986)	<i>Product and addition formulas for the continuous <math>q</math>-ultraspherical polynomials</i>	SIAM Journal on Mathematical Analysis
Askey et al. (1986)	<i>Limits of some <math>q</math>-Laguerre polynomials</i>	Journal of Approximation



		Theory
Ismail et al. (1986)	<i>Asymptotics of the Askey-Wilson and <math>q</math>-Jacobi polynomials</i>	SIAM Journal on Mathematical Analysis
Gaspar et al. (1986)	<i>Positivity of the Poisson kernel for the continuous <math>q</math>-Jacobi polynomials and some quadratic transformation formulas for basic hypergeometric series</i>	SIAM Journal on Mathematical Analysis
R. Askey (1987)	<i>An integral of Ramanujan and orthogonal polynomial</i>	Journal of the Indian Mathematical Society
Al-Salam et al. (1987)	<i><math>q</math>-Pollaczek polynomials and a conjecture of Andrews and Askey</i>	SIAM Journal of Mathematical Analysis
Ismail et al. (1987)	<i>The combinatorics of <math>q</math>-Hermite polynomials and the Askey-Wilson integral</i>	European Journal of Combinatorics
J.Wimp (1987)	<i>Explicit formulas for the associated Jacobi polynomials and some applications.</i>	Canadian Journal of Mathematics
S.N.Singh (1987)	<i>Certain transformation of Abnormal basic hypergeometric functions</i>	Ramanujan International Symposium on analysis
S.N.Singh (1987)	<i>On <math>q</math>-series and continued fraction</i>	Proc. Math. Soc., B.H.U.
M. Rahman (1988)	<i>A projection formula for the Askey-Wilson polynomials and an application</i>	Proceedings of the American Mathematical Society
Al-Salam et al. (1988)	<i><math>q</math>-Beta integrals and the <math>q</math>-Hermite polynomials</i>	Pacific Journal of Mathematics
Ismail et al. (1988)	<i>On the Askey-Wilson and Rogers polynomials</i>	Canadian Journal of Mathematics
N.A. Bhagirathi (1988)	<i>Certain investigations in the field of generalized basic hypergeometric functions and continued fractions</i>	Ph. D. Thesis, University of Gorakhpur
M. Rahman (1989)	<i>A simple proof of Koornwinder's addition formula for the little <math>q</math>-Legendre polynomials</i>	Proceedings of the American Mathematical Society
R. Askey (1989)	<i>Beta integrals and the associated orthogonal polynomials</i>	In: Number Theory (ed. K. Alladi). Lecture Notes in Mathematics 1395, Springer-Verlag, New York
R. Askey (1989)	<i>Divided difference operators and classical orthogonal polynomials.</i>	The Rocky Mountain Journal of Mathematics
Ismail et al.(1989)	<i>Quadratic birth and death processes and associated continuous dual Hahn polynomials.</i>	SIAM Journal on Mathematical Analysis
T.H. Koornwinder (1989)	<i>Continuous <math>q</math>-Legendre polynomials as spherical matrix elements of irreducible representations of the quantum <math>SU(2)</math> group.</i>	C.W.I. Quarterly
T.H. Koornwinder (1989)	<i>Representations of the twisted <math>SU(2)</math> quantum group and some <math>q</math> hypergeometric orthogonal polynomials.</i>	Indagationes Mathematicae
Srivastava et al. (1989)	<i>Some multilinear generating functions for <math>q</math>-Hermite polynomials</i>	Journal of Mathematical Analysis and Applications
G. Gaspar (1989)	<i><math>q</math>-Extensions of Clausen's formula and of the inequalities used by De Branges in his proof of the Bieberbach, Robertson, and Milin conjectures.</i>	SIAM Journal on Mathematical Analysis

Gasper et al. (1989)	<i>A non-terminating <math>q</math>-Clausen formula and some related product formulas.</i>	SIAM Journal on Mathematical Analysis
Bustoz et al.(1989)	<i>A positive trigonometric sum.</i>	SIAM Journal on Mathematical Analysis
T.H. Koornwinder(1990)	<i>Jacobi functions as limit cases of <math>q</math>-ultraspherical polynomials</i>	Journal of Mathematical Analysis and Applications.
Srivastava et al. (1990)	<i>Some formulas involving <math>q</math>-Jacobi and related polynomials</i>	Annali di Matematica Pura ed Applicata
D. Stanton (1990)	<i>An introduction to group representations and orthogonal polynomials.</i>	In: Orthogonal Polynomials: Theory and Practice (ed. P. Nevai), Kluwer Academic Publishers, Dordrecht
Stanton et al. (1990)	<i>An introduction to group representations and orthogonal polynomials.</i>	In: Orthogonal Polynomials: Theory and Practice (ed. P. Nevai), Kluwer Academic Publishers, Dordrecht.
Gasper et al. (1990)	<i>Basic Hypergeometric Series. Encyclopedia of Mathematics and Its Application</i>	Cambridge University Press, Cambridge
S.N.Singh (1990)	<i>An expansion involving basic hypergeometric functions</i>	J. P.A.S.
Rahman et al.(1991)	<i>Positivity of the Poisson kernel for the Askey-Wilson polynomials</i>	Indian Journal of Mathematics
Ismail et al. (1991)	<i>Complex weight functions for classical orthogonal polynomials</i>	Canadian Journal of Mathematics
Ismail et al. (1991)	<i>The associated Askey-Wilson polynomials.</i>	Transactions of the American Mathematical Society
R.P.Agarwal (1991)	<i>Ramanujan's last gift</i>	Math, Student
Jain et al. (1992)	<i>New results involving a certain class of <math>q</math>-orthogonal polynomials</i>	Journal of Mathematical Analysis and Applications
R.P.Agarwal (1992)	<i>Pade approximants, Continued fractions and Heine's <math>q</math>-hypergeometric series</i>	Jour. Math. Phy. Sci.
R.P.Agarwal (1993)	<i>Lambert series and Ramanujan</i>	Proc. Indian Acad. Sci. (Math Sci.)
U.B.Singh (1993)	<i>On the sums of certain basic bilateral hypergeometric series</i>	Bull Cal. Math. Soc
M.E.H. Ismail (1994)	<i>Asymptotics of Pollaczek polynomials and their zeros</i>	SIAM Journal on Mathematical Analysis
Ismail et al. (1994)	<i><math>q</math>-Hermite polynomials, biorthogonal rational functions, and <math>q</math>-beta integrals</i>	Transactions of the American Mathematical Society
Ismail et al. (1994)	<i>Diagonalization of certain integral operators</i>	Advances in Mathematics
Jain et al. (1995)	<i>Some families of multilinear <math>q</math>-generating functions and combinatorial <math>q</math>-series identities.</i>	Journal of Mathematical Analysis and Applications
M. Rahman (1996)	<i>Some generating functions for the associated Askey-Wilson polynomial</i>	Journal of Computational and Applied Mathematics
Gupta et al. (1996)	<i>Contiguous relations, basic hypergeometric functions, and orthogonal polynomials III. Associated continuous dual <math>q</math>-Hahn polynomials.</i>	Journal of Computational and Applied Mathematics
H. Exton (1996)	<i>New generating functions for Gegenbauer polynomials.</i>	Journal of Computational and Applied Mathematics

Ismail et al.(1997)	<i>Some generating functions for <math>q</math> polynomials. In: Special Functions, <math>q</math>-Series and Related Topics (eds. M.E.H. Ismail, D.R. Masson and M. Rahman).</i>	Fields Institute Communications
Ismail et al. (1998)	<i>The <math>q</math>-Laguerre polynomials and related moment problems</i>	Journal of Mathematical Analysis and Applications
H. Exton (1998)	<i>Summation formulae involving the Laguerre polynomial.</i>	Journal of Computational and Applied mathematics
Andrews et al. (1999)	<i>Special Functions. Encyclopedia of Mathematics and Its Applications</i>	Cambridge University Press, Cambridge
G. Bangerezako (1999)	<i>The factorization method for the Askey-Wilson polynomials.</i>	Journal of Computational and Applied Mathematics
Atakishiyev et al. (2004)	<i>On <math>q</math>-orthogonal polynomials, dual to little and big <math>q</math>-Jacobi polynomials.</i>	Journal of Mathematical Analysis and Applications
M.E.H. Ismail (2005)	<i>Asymptotics of <math>q</math>-orthogonal polynomials and a <math>q</math>-Airy function</i>	International Mathematics Research Notices
K.W.J. Kadell (2005)	<i>The little <math>q</math>-Jacobi functions of complex order. In: Theory and Applications of Special Functions.</i>	Developments in Mathematics 13, Springer, New York
T.H. Koornwinder (2005)	<i>A second addition formula for continuous <math>q</math>-ultraspherical polynomials</i>	In: Theory and Applications of Special Functions. Developments in Mathematics 13, Springer, New York
Datta et al. (2006)	<i>A characterization of some <math>q</math>-orthogonal polynomials</i>	Ramanujan Journal
Atakishiyev et al. (2007)	<i>The factorization of a <math>q</math>-difference equation for continuous <math>q</math>-Hermite polynomials. Journal of Physics.</i>	A. Mathematical and Theoretical
Atakishiyeva et al. (2008)	<i>On continuous <math>q</math>-Hermite polynomials and the classical Fourier transform.</i>	Journal of Physics A. Mathematical and Theoretical

#### IV. CONCLUSIONS AND FUTURE WORK

$q$  method has very broad spectrum of applications. It is used in fields like solid state theory, mechanical engineering, operational calculus, quantum theory, cosmology, Lie theory, linear algebra, high energy particles physics, Fourier Analysis, elliptic functions etc.

Some of the fields where it has a wide scope are listed below.

1. Numerical solutions to differential equations for boundary value analysis
2. Finite Difference Method
3. Computational Fluid Dynamics (Navier–Stokes Equations)
4. Dynamics (Newton-Euler & Lagrange's equations)
5. Finite Element Method
6. Solid Mechanics (Elasticity equations)
7. Heat Transfer (Heat equation)
8. Kinematics Simulation
9. Complex System Optimization
10. DNA Analysis
11. Computer Graphics Theory

12. Finger Print Verification
13. Signal Processing
14. Air Craft Designing

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