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On Second Degree Equation with Three Unknowns $5(x^2+y^2)-9xy=100z^2$

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ABSTRACT:
The cone represented by the ternary quadratic Diophantine equation $5(x^2+y^2)-9xy=100z^2$ *is analyzed for its patterns of non-zero distinct integral solutions.*
KEYWORDS: *Ternary quadratic, homogeneous quadratic, integral solutions*

I. INTRODUCTION:

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-15] for quadratic equations with three unknowns. This communication concerns with yet another interesting equation $5(x^2+y^2)-9xy=100z^2$ representing homogeneous quadratic with three unknowns for determining its infinitely many non-zero integral points.

II. METHOD OF ANALYSIS:

The ternary quadratic diophantine equation to be solved for its non-zero distinct integral solution is

$$5(x^2+y^2)-9xy=100z^2 \tag{1}$$

We present below different methods of solving (1) and obtain different sets of solutions to (1).

Set I:

Introduction of the linear transformations

$$x=(u+v),\ y=(u-v),\ u\neq v\neq 0 \tag{2}$$

in (1) leads to

$$u^2+19v^2=100z^2 \tag{3}$$

We can write 100 as

$$100=(9+i\sqrt{19})(9-i\sqrt{19}) \tag{4}$$

Assume

$$z=a^2+19b^2 \tag{5}$$

where a and b are non-zero distinct integers.

Using (4) & (5) in (3) and employing the method of factorization , consider

$$(u+i\sqrt{19}v)=(9+i\sqrt{19})(a+i\sqrt{19}b)^2 \tag{6}$$

Equating the real and imaginary parts either in (6) , we get

$$u=9(a^2-19b^2)-38ab,$$

$$v=(a^2-19b^2)+18ab$$

Substituting the values of u and v in (2) ,we have

$$x=10(a^2-19b^2)-20ab,$$

$$y=8(a^2-19b^2)-56ab$$

which satisfy (1) along with (5).

Set II:

Introduction of the linear transformations

$$x = 10(u+v),\ y = 10(u-v),\ u \neq v \neq 0 \tag{7}$$

in (1) leads to

$$u^2 + 19v^2 = z^2 \tag{8}$$

which is satisfied by

$$v = 2rs,\ u = 19r^2 - s^2$$

and

$$z = 19r^2 + s^2 \tag{9}$$

Substituting the values of u and v in (7) ,we have

$$x = 10(19r^2 - s^2 + 2rs),$$

$$y = 10(19r^2 - s^2 - 2rs)$$

which satisfy (1) along with (9).

Set III:

Write (8) as the system of double equations as in Table 1 below:

System	1	2	3
$z+u$	$19v^2$	v^2	$19v$
$z-u$	1	19	v

Solving each of the above systems ,the values of u ,v ,z are obtained. In view of (7), the corresponding values of x ,y are found. For simplicity ,the integer solutions thus obtained are exhibited below:

Solutions from system I:

$$x = 10(38k^2 + 40k + 10),$$

$$y = 10(38k^2 + 36k + 8),$$

$$z = (38k^2 + 38k + 10)$$

Solutions from system II:

$$x = 10(2k^2 + 4k - 8),$$

$$y = 10(2k^2 - 10),$$

$$z = (2k^2 + 2k + 10)$$

Solutions from system III:

$$x = 100k,$$

$$y = 80k,$$

$$z = 10k$$

Set IV:

Equation (8) is written as

$$u^2 + 19v^2 = z^2 * 1 \tag{10}$$

Write 1 on the R.H.S. of (10) as

$$1 = \frac{(5 + i3\sqrt{19})(5 - i3\sqrt{19})}{196} \tag{11}$$

Using (5) & (11) in (10) and employing the method of factorization ,consider

$$(u + i\sqrt{19}v) = \frac{(5 + i3\sqrt{19})}{14}(a + i\sqrt{19}b)^2$$

Equating real and imaginary parts in the above equation ,the values of u,v are obtained. Replacing a by 7A, b by 7B in the above resulting values of u,v and (5) ,we get

$$x = 70(4A^2 - 76B^2 - 52AB),$$

$$y = 70(A^2 - 19B^2 - 62AB),$$

$$z = 49(A^2 + 19B^2)$$

which represent the solutions to (1).

Note: 1

It is seen that 1 on the R.H.S. of (10) is also represented as follows:

$$1 = \frac{(9 + i\sqrt{19})(9 - i\sqrt{19})}{100},$$

$$1 = \frac{(19r^2 - s^2 + i\,2rs\sqrt{19})(19r^2 - s^2 + i\,2rs\sqrt{19})}{(19r^2 + s^2)^2}$$

Following the above procedure, two more sets of integer solutions to (1) are obtained.

Set V:

Equation (8) is written as

$$z^2 - 19v^2 = u^2 * 1 \qquad (12)$$

Assume

$$u = a^2 - 19b^2 \qquad (13)$$

Write 1 on the R.H.S. of (12) as

$$1 = \frac{(14 + 3\sqrt{19})(14 - 3\sqrt{19})}{25} \qquad (14)$$

Following the analysis as in Set **IV ,**the corresponding integer solutions

to (1) are given by

$$x = 50(8a^2 - 38b^2 + 28ab),$$

$$y = 50(2a^2 - 152b^2 - 28ab).$$

$$z = 5(14(a^2 + 19b^2) + 114ab)$$

Note: 2

It is seen that 1 on the R.H.S. of (12) is also given by

$$1 = \frac{(10 + \sqrt{19})(10 - \sqrt{19})}{81},$$

$$1 = \frac{(19r^2 + s^2 + 2rs\sqrt{19})(19r^2 + s^2 - 2rs\sqrt{19})}{(19r^2 - s^2)^2}$$

Following the above procedure,two more sets of solutions to (1) are obtained.

III. CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of homogeneous ternary quadratic equations .

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